Macro Theory B

Final exam (spring 2015)

A suggested solution

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1 Incomplete markets versus central planner

1. The state variables for the household are the levels of assets saved from the previous period $a_{i,t-1}$ and the parameter for preference for leisure $\psi_{i,t}$.

The household problem:

$$\begin{aligned} V\left(a_{t-1},\psi_t\right) &= \max_{c_t,h_t,a_t}\left\{u\left(c_t\right)+\psi_t\left(1-h_t\right)+\beta E_t V\left(a_t,\psi_{t+1}\right)\right\} \\ & s.t. \\ a_t &\geq 0 \\ c_t+a_t &\leq w_t h_t+(1+r_{t-1})a_{t-1} \end{aligned}$$

and the stochastic process of ψ

The only aggregate state variable is K_t.

2. The stationary recursive equilibrium is:

- A set of prices w,r
- A decision function for the households $[c_t,h_t,a_t]=g\left(a_{t-1},\psi_t,r,w\right)$
- A decision function for the firms how much capital and labor to employ
- A distribution of capital among households, for every possible state of ψ

Such that:

- The household decision function is optimal given the prices and the constrains
- The firm decision function is optimal given the prices
- The aggregate resource constrain holds: $C_t+K_{t+1}-(1-\delta)K_t=AK_t^{\alpha}H_t^{1-\alpha}$
- The capital distribution is stationary

3. An algorithm to calculate the equilibrium:

1. Guess an initial interest level r
2. From the firm's optimal decision level for capital $r=\alpha A(K/H)^{\alpha-1}$ calculate the capital/labor ratio. Then use it to calculate the wage from the firm's optimal decision for labor $w=\left(1-\alpha\right)A\left(K/H\right)^{\alpha}$
3. Solve the household problem:
   (a) Define a grid for the assests per each leisure type
   (b) Guess an initial value vector per type $V\left(a,\psi\right)$

(c) For each type and for each asset level, find the best next asset level by solving the household problem defined in section 1. Note that this involves using the FOC for work hours, as if you have an asset level and want to move to a different asset level, there is a unique combination of c, h that will bring you there while obeying the FOC

(d) Iterate until convergence

4. Find a stationary distribution for the household assets:

   (a) Using the household solution to build the decision rule for next period assets

   (b) Building the transition matrixes from current level of assets to next period levels

   (c) Iterating until convergence

5. Calculate total capital and labor supply from the stationary distribution and the household decision rule

6. Calculate total capital and labor demand using the firm's FOC

7. Update the guess for r using capital-labor ration demand and supply, iterate (2)-(6) until capital demand and supply converge

4. The objective goal of the central planner is:

$$\begin{aligned} & \max_{c_i,h_i} \int \left[u\left(ci\right) + \psi_i\left(1-h_i\right)\right] \\ & s.t. \\ C_t + K_{t+1} - (1-\delta)K_t &= AK_t^{\alpha}H_t^{1-\alpha} \end{aligned}$$

The central planner maximizes the (potentially weighted) sum of the housseholds' welfare. As the central planner decides about the level of consumption and leisure per each household, the assest distribution is immatirial, and the only state variable is the aggregate capital K. The central planner is only constrained by the aggregate production function. Note that as the central planner is making all the consumption and work decisions, there is no relevance for the individual ownership of capital, and the capital distribution doesnt matter at all.

5. We saw in class that in the IM economy, as households cannot insure themselves against idiosyncratic shock (of preferences, in this case), they will "self-insure" by saving. As the central planner can insure households by allocating the consumption at will, there will be less need for savings in the economy, and the consumpation path will be more smooth. Also, as the central planner is maximizing total (weighted) utility, she will equate the marginal utility from consumption (i.e. consumption of all households will be equal if the CP assigns the same weight to all) and will also equate the marginal (dis)utility from work, given ψ_i

2 An Island model with a twist

1. A worker moves only if the (expected) value from being in the other island in period 3 is higher than being on your own island in period 3, and the difference should be high enough to compensate for the loss of wages in periods1 and 2, so there will only be movement in one direction. Given that $\pi > \frac{1}{2}$, if no one moves, the value in island 2 in period 3 has to be higher than the value island 1 in period 3, so if there is movement, it has to be from island 1 to island 2.

2. The value functions in period 1 in islands 1 and 2, assuming x workers have left island 1:

$$\begin{aligned} V_1^1 &= (1+\beta)\,\theta_L f'(1/2 - x) + \beta^2 V_1^3 \\ V_2^1 &= (1+\beta)\,\theta_H f'(1/2) + \beta^2 V_2^3 \end{aligned}$$

If the workers will actually continue to island 2, than in period 3 there will be more workers in island 2 than in island 1. This is beneficial for the moving workers only if the productivity level in island 2 is indeed higher than in island 1. The 3rd period productivity level is revealed in period 2, when there is still time to return. From all possible states of nature, only the state in which both islands retian their original productivity level is a state where the productivity in period 3 is higher in island 2. Lets consider all possible states of nature in period 3:

	island 1	island 2	probability	Movers go to
1	θ_L	θ_H	π^2	Island 2
2	θ_L	θ_L	$\pi(1-\pi)$	Island 1
3	θ_H	θ_H	$(1-\pi)\,\pi$	Island 1
4	θ_H	θ_L	$(1-\pi)^2$	Island 1

The moving workers will continue to island 2 only in state 1, and will return in all other states. So, we can now write the value functions in period 3:

$$\begin{aligned} V_1^3 &= \pi^2 \theta_L f'(1/2 - x) + [\pi(1-\pi)\,\theta_L + (1-\pi)\,\theta_H]\, f'(1/2) \\ V_2^3 &= \pi^2 \theta_H f'(1/2 + x) + [(1-\pi)\,\theta_L + (1-\pi)\,\pi\theta_H]\, f'(1/2) \end{aligned}$$

3. A moving worker increases her chance to be on a high productivity island from $(1-\pi)$ if she stays at island 1, to $1-\pi(1-\pi)$ if she moves in period 1. This is beneficial for the first worker that is considering to move only if:

$$\beta^2 \left\{ \begin{array}{c} \pi(1-\pi)\,\theta_L + \\ [1-\pi(1-\pi)]\,\theta_H \end{array} \right\} f'(1/2) >= (1+\beta)\,\theta_L f'(1/2) + \beta^2 \left\{ \begin{array}{c} \pi\theta_L + \\ (1-\pi)\,\theta_H \end{array} \right\} f'(1/2)$$

4. In Equilibrium the marginal worker in island 1 is indifferent between staying and moving. The value of moving is:

$$\beta^2\left\{\pi^2\theta_H f^{'}\left(1/2+x\right)+\left[\pi\left(1-\pi\right)\theta_L+\left(1-\pi\right)\pi\theta_H\right]f^{'}\left(1/2\right)\right\}$$

where the first term in the brackets is in case the employee ends up on island 2 and the second term in case she returns to island 1. So the conditon is:

$$\beta^2\left\{\pi^2\theta_H f^{'}\left(1/2+x\right)+\left[\pi\left(1-\pi\right)\theta_L+\left(1-\pi\right)\pi\theta_H\right]f^{'}\left(1/2\right)\right\}=V_1^3$$

3 Market completeness

1. This is a complete market economy as there is a market for each good in each state

2. The market clearing condition is that the amount of resources paid to the holders of the asset that pays if the agent is unemployed, $(1-\pi)\,b_u$, is equel to the amount of resources paid by the holders of he asset that pays when the holder is employed, $-\pi b_e$. Thus, the market clearing price has to be π.

3. Aggregate consumption is the amount consumed by the unemployed, $(1-\pi)\,b_u$, plus the amount consumed by the employed, $\pi w-\pi b_e$. So, Aggregate consumption is:

$$(1-\pi)\,b_u+\pi w-\pi b_e=\pi w$$

and equel to aggregate resources